

What nuclear reactions imply for astrophysical processes

R. Ogul^{1,2}, N. Buyukcizmeci ^{1,3}, A. S. Botvina^{3,4}, I. N. Mishustin^{3,5}, W. Trautmann²

¹Department of Physics, Selcuk University, 42075 Konya, Turkey

²GSI GmbH, D-64291 Darmstadt, Germany

³FIAS, J. W. Goethe University, D-60438 Frankfurt am Main, Germany

⁴Institute for Nuclear Research RAS, 117312 Moscow, Russia

⁵Kurchatov Institute, Russian Research Center, 123182 Moscow, Russia

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Nuclear matter & Stellar matter

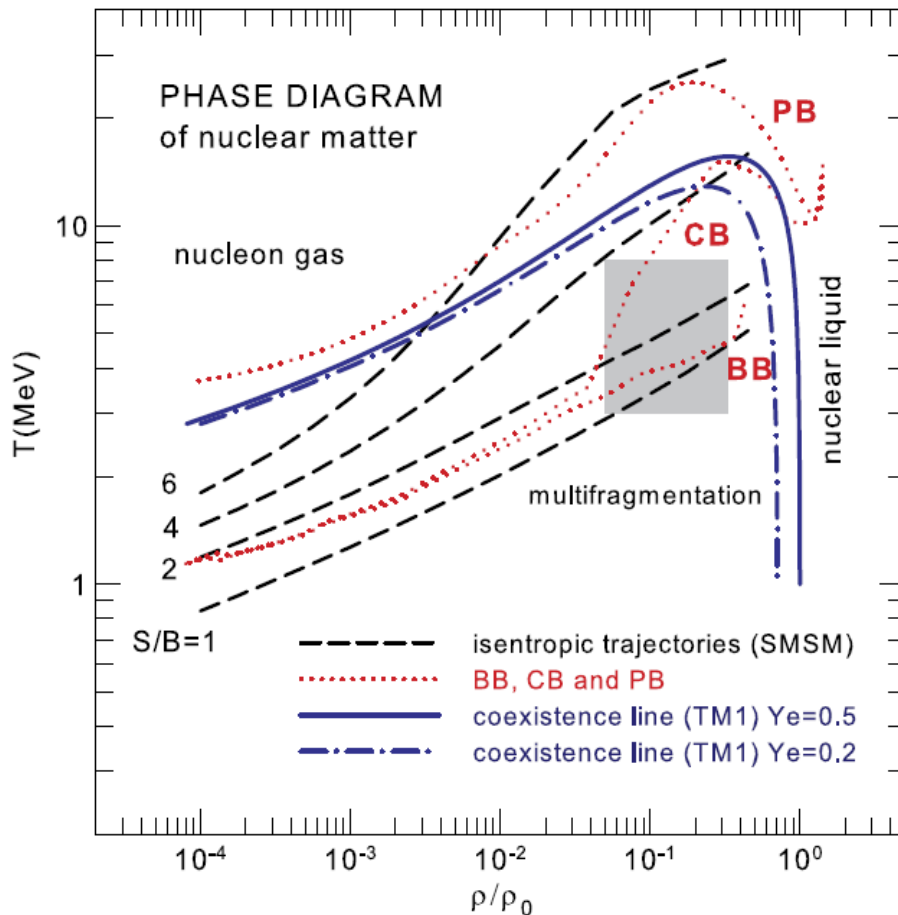


Fig.1. Nuclear EOS in $T - \rho$ plane, solid lines: $Y_e=0.5$ dashed-dotted: $Y_e=0.2$, PB: post bounce, CB: at the core bounce, BB: before bounce in a core-collapse supernova. $T=0.5-10$ MeV, $\rho/\rho_0=10^{-1}, 10^{-2}, 10^{-3}$, $Y_e=0.2-0.4$

Ref: N. Buyukcizmeci et al, NPA 907 (2013)13-54.

Nuclear matter: liquid-gas type phase trans in finite nuclei, formation of hot fragments at low density freeze-out, $T \approx 3-8$ MeV, and $\rho \approx (0.1 - 0.4)\rho_0$.

Stellar matter: at subnuclear densities expected during the collapse and subsequent explosions of massive stars (when the core of massive stars collapse in a burst of electron capture high densities reached then repulsive NN interactions give rise to Shock waves which is responsible for supernova explosion). Hydrodynamical simulations at

$T \sim 0.5 - 10$ MeV, and $\rho \approx (0.001 - 0.1)\rho_0$

To establish a relationship: we discuss similarities & differences of conditions in supernova explosions and heavy-ion collisions. In Statistical Model for Supernova Matter (SMSM), we consider nuclear, em, and weak reactions between all constituents of stellar matter in equilibrium. Surrounding species, such as electrons, neutrinos produce significant changes in binding energies and decay modes.

Nuclear dynamics from low to relativistic energies:

Terrestrial labs to investigate the properties of rare isotopes and nuclear reactions with **stable-ion beams** (LINAC/SPIRAL2-Ganil, ...) and **radioactive ion-beams (RIB)** (GSI-FAIR, DUBNA, GANIL, CATANIA, TAMU, RIKEN, ...).

In nature, these reactions would take place in **stars** and exploding stellar environments such as **novae** and **supernovae**.

Research on nuclear structure, astrophysics, **nuclear dynamics from low to relativistic energies**, fundamental interactions, and so on...

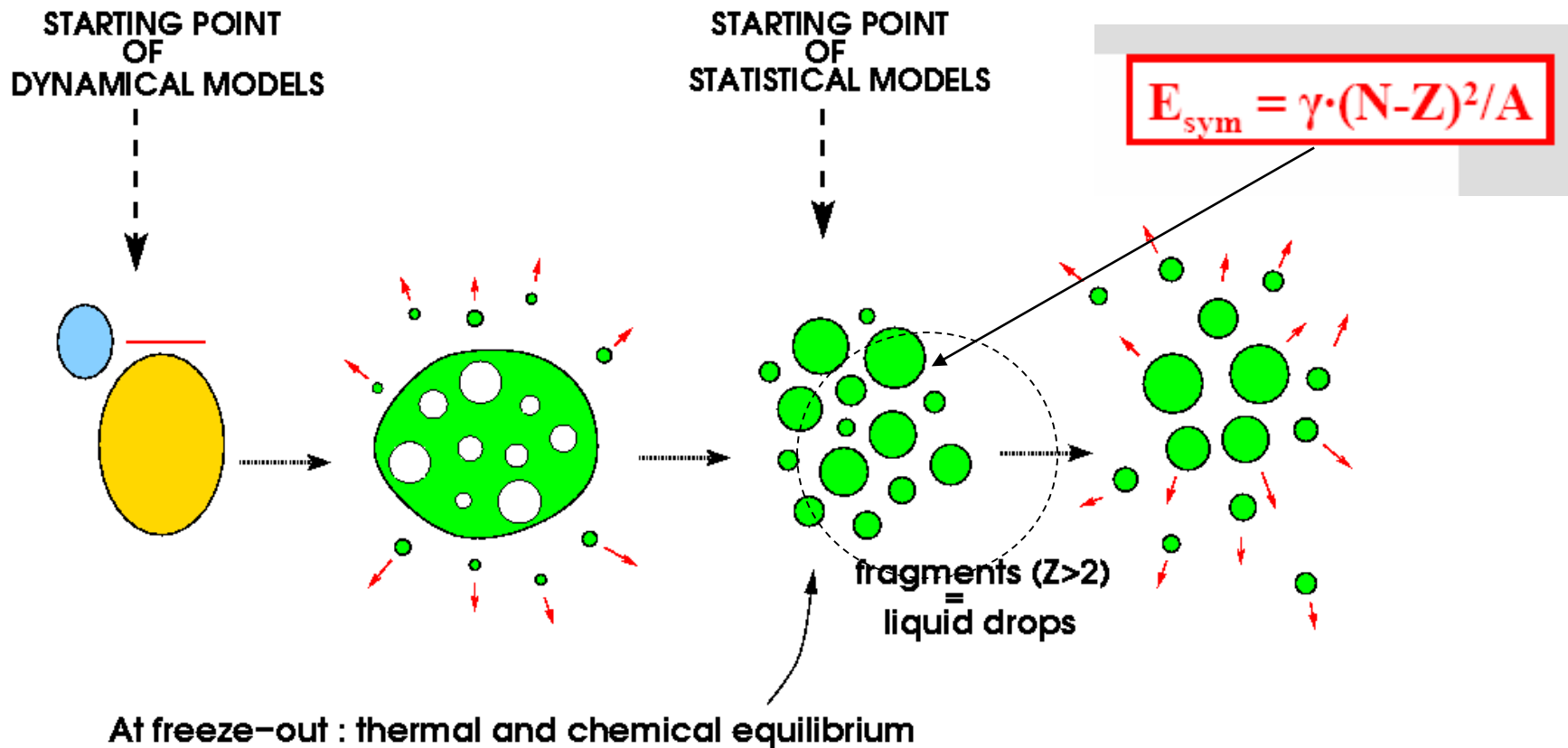
Nuclear matter **EOS (symmetry energy)** is crucial in simulation of astrophysical events such as core-collapse supernovae and neutron stars.

Symmetry energy effects: Theoretical studies to predict the density dependence of symmetry energy needs experimental data at various energies with a diversity of isospin components of the reaction systems.

Multifragmentation in intermediate and high energy heavy-ion collisions

Experimentally established: 1) few stages of reactions leading to multifragmentation, 2) short time $\sim 100\text{fm}/c$ for primary fragment production, 3) freeze-out density is around $0.1\rho_0 - 0.4\rho_0$, 4) high degree of equilibration at the freeze-out.

formation of an equilibrated nucl. system \rightarrow disassembly of the system \rightarrow deexcitation



Statistical multifragmentation model (SMM)

A compressed and hot blob of nuclear matter is formed in the collision of heavy ions. Assuming this matter to be close to thermodynamic eq. it will expand and enter the region of subnuclear densities. It is assumed in SMM that a statistical equilibrium is reached in the low density freeze-out region (t scales > 1000 fm/c at $T < T_c$ sufficient time to reach thermal and chemical equilibrium).

The breakup channels are composed of nucleons and nuclear fragments, the laws of conservation of energy, momentum, angular momentum, A , and Z are considered.

The compound-nucleus channels such as evaporation and fission processes at low excitation energies ($E^* \leq 1$ MeV/nucleon, Weisskopf evaporation) as well as the transition region between the low- and the high-energy de excitation regimes are also included, and competition among all channels is permitted.

In the thermodynamic limit, SMM is consistent with liquid-gas phase transitions when the liquid phase is represented by infinite nuclear clusters [1], which allow connections for the astrophysical studies [2].

We calculate the statistical weights of all breakup channels. The decay channels are generated by the Monte Carlo method according to their statist weights. In the Markov chain SMM [3] we use the ingredients of standard SMM. Free energy of the fragments are parametrized as

$$F_{A,Z} = F_{A,Z}(\text{bulk}) + F_{A,Z}(\text{surface}) + F_{A,Z}(\text{coul.}) + F_{A,Z}(\text{symm.})$$

Statistical multifragmentation model (SMM)

probability of a given partition: $W_f^{mic} = \frac{1}{\xi} \exp \{S_f(E_0, A, Z)\}$, with $\xi = \sum_f \exp \{S_f(E_0, A, Z)\}$

mass and charge conservation: $\sum_{A,Z} N_{AZ} \cdot A = A_0$, $\sum_{A,Z} N_{AZ} \cdot Z = Z_0$

energy conservation: $E_0 = F_f - T_f \frac{\partial F_f}{\partial T_f}$

entropy of channel: $S_f = -\frac{\partial F_f}{\partial T_f}$

Fragments obey Boltzmann statistics, liquid-drop description of individual fragments, Coulomb interaction in the Wigner-Seitz approximation

free energy of a channel: $F_f = \sum_{A,Z} N_{AZ} F_{AZ} + \frac{3}{5} \frac{Z_0^2 e^2}{r_0 A_0^{1/3} (1 + \chi_c)^{1/3}}$

individual fragments: $F_{AZ} = F_{AZ}^b + F_{AZ}^s + F_{AZ}^{sym} + F_{AZ}^c$

bulk term: $F_{AZ}^b = [-W_0 - T_f^2 / \varepsilon_0(A)]A$, $W_0 = 16 \text{ MeV}$

surface term: $F_{AZ}^s = B_0 [(T_c^2 - T_f^2) / (T_c^2 + T_f^2)]^{5/4} A^{2/3}$, $B_0 = 18 \text{ MeV}$

symmetry term: $F_{AZ}^{sym} = \gamma(N - Z)^2 / A$, $T_c = 18 \text{ MeV}$

Coulomb term: $F_{AZ}^c = \frac{3}{5} \frac{Z_0^2 e^2}{r_0 A_0^{1/3}} [1 - (1 + \chi_c)^{-1/3}]$, $\gamma = 25 \text{ MeV}$

Analysis of ALADIN Experiments (GSI)

ELEMENT	A	Z	N	N/Z
^{124}Sn	124	50	74	1.48
^{124}La	124	57	67	1.18
^{107}Sn	107	50	57	1.14

Table 1. The neutron content of the nuclei affects the fragment production in multifragmentation of finite nuclei.

For our calculations, we consider 3 different sources $_{50}\text{Sn}^{124}$ neutron-rich, and $_{57}\text{La}^{124}$, $_{50}\text{Sn}^{107}$ neutron-poor to see how isospin effects the multifragmentation phenomena with their proton to neutron ratios (N/Z) are 1.48, and 1.18, 1.14, respectively. All beams had a laboratory energy of 600 MeV/nucleon and were directed onto reaction targets consisting of natSn with areal density 500 or 1000 mg/cm².

Isotopic yields for light particles ($Z \leq 10$) and IMF have been measured and calculated within SMM-ensemble by means of isoscaling, isotopic curves and N/Z to address open questions about modifications of symmetry and surface terms for hot fragments Ref. [1].

[1]. R. Ogul, A.S. Botvina et al., Phys. Rev. C 83, 024608 (2011).

Ensemble of sources

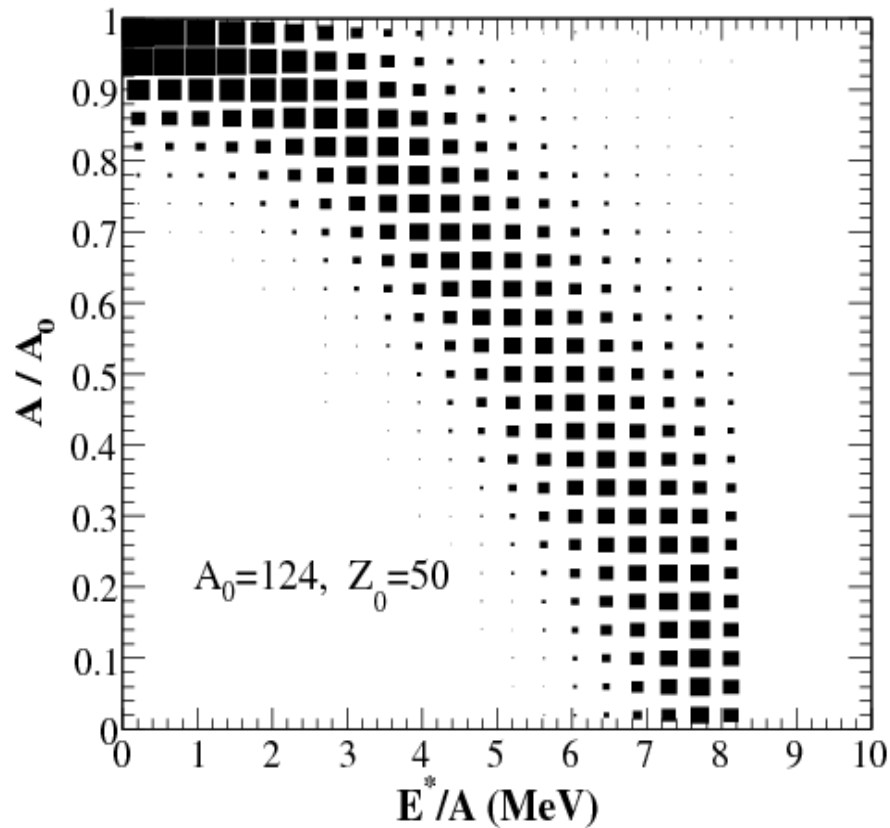
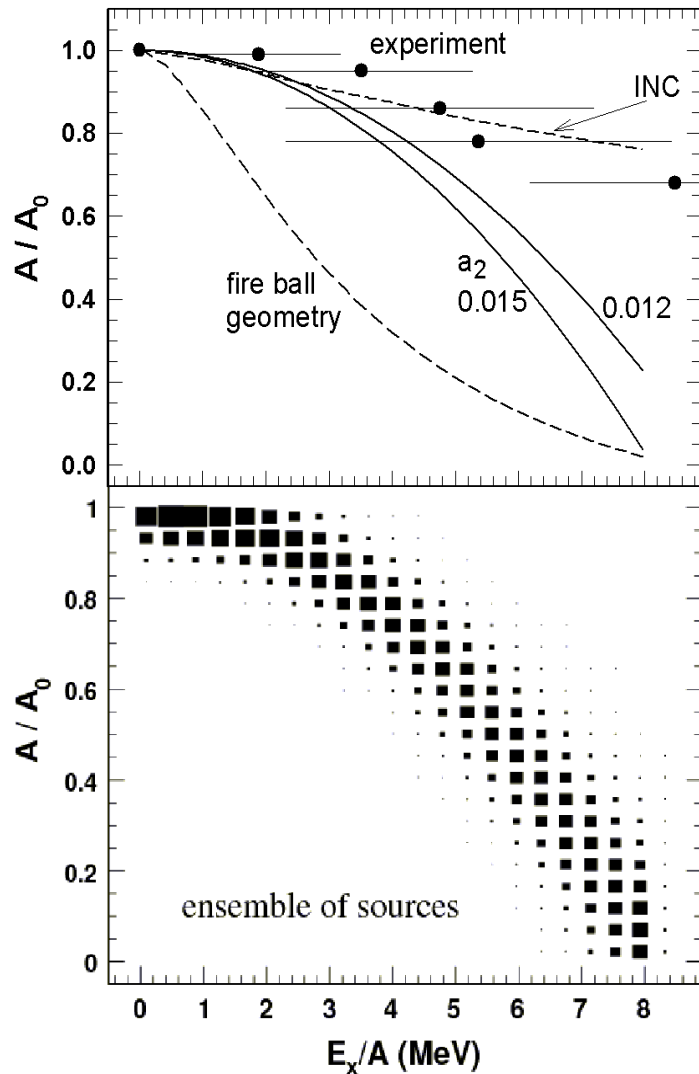


Fig.1. Ensemble of hot thermal sources represented in a scatter plot of reduced mass number A/A_0 of thermal sources as a function of their excitation energy E^*/A , as used in the SMM ensemble calculations. The intensity of the individual sources is proportional to the area of the squares.

The hot sources are formed according to the impact parameters. In this work, we follow Ref.[2], which was rather effective in describing the multifragmentation of relativistic ^{197}Au projectiles, including their correlations and dispersions. The average masses of the equilibrated sources A were parametrized as $A/A_0 = 1 - a_1(\text{Ex}/A_s) - a_2(\text{Ex}/A_s)^2$, where Ex is the excitation energy per nucleon of the sources in MeV and A_0 is the projectile mass. According to the parametrization used in this work for $a_2 = 0.012$ and 0.015 .

[2]. A. S. Botvina, et al., Phys. At. Nucl. 57, 628 (1994).

Statistical ensemble of sources in SMM (ALADIN exps-GSI)



R. Ogul, A.S. Botvina et al., Phys. Rev. C 83, 024608 (2011).

Fig.2. Top panel: Mean reduced mass number A/A_0 of relativistic projectile residuals as a function of their excitation energy E_x/A according to the fireball model of Gosset et al. [1], calculated for ^{107}Sn projectiles, and INC calculations for $^{197}\text{Au} + \text{C}$ [2] (dashed lines), according to the experimental results for ^{197}Au nuclei reported by Pochodzalla et al. [3] and including the widths in E_x/A (dots) and according to the parametrization used in this work for $a_2 = 0.012$ and 0.015 MeV^{-2} (solid lines).

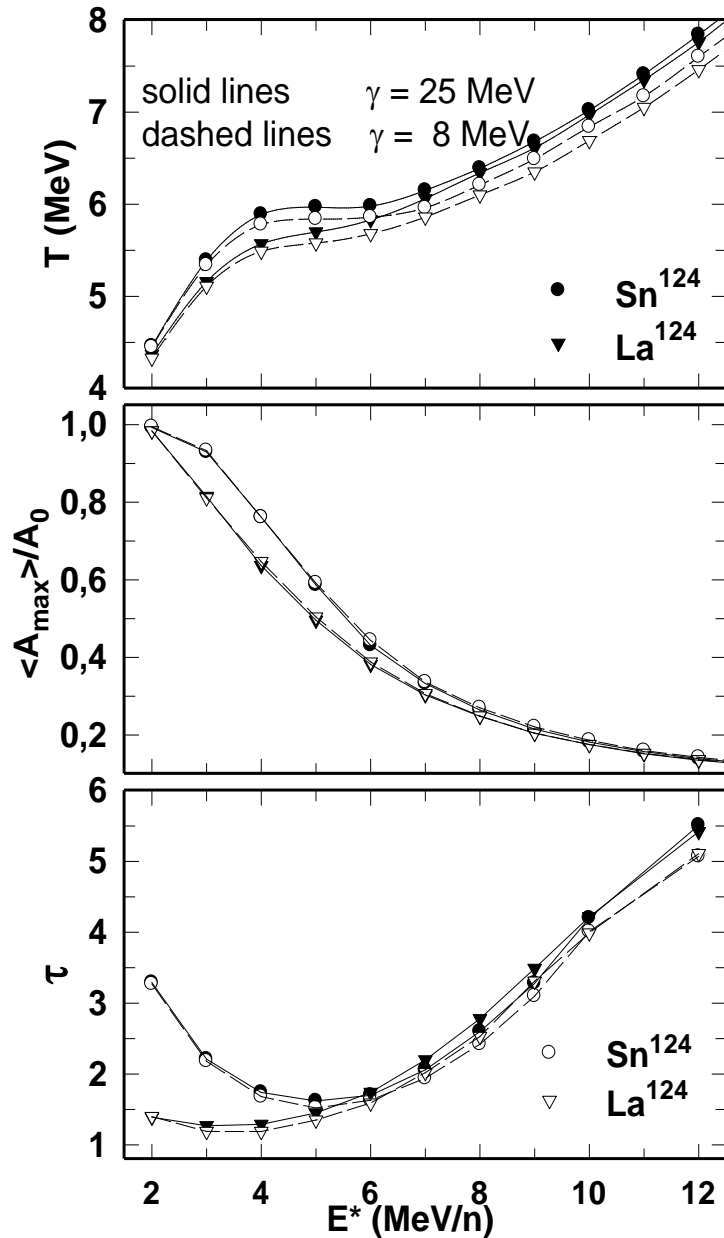
Bottom panel: Ensemble of hot thermal sources represented in a scatter plot of reduced mass number A/A_0 versus excitation energy E_x/A , as used in the SMM calculations. The frequency of the individual sources is proportional to the area of the squares.

[1]. J. Gosset, et al., Phys. Rev. C 16, 629 (1977).

[2]. A. S. Botvina, et al., Phys. At. Nucl. 57, 628 (1994).

[3]. J. Pochodzalla et al., Phys. Rev. Lett. 75, 1040 (1995).

Influence of the symmetry energy on average characteristics



$$F_{AZ}^{sym} = \gamma (N - Z)^2 / A$$

The calculations for $\gamma = 25$ MeV corresponding to the mass formula of cold nuclei, and for $\gamma = 8$ MeV to hot sources Sn^{124} and La^{124} (isolated nuclei or in-medium).

The caloric curve, A_{\max} the mean maximum mass of fragments in partitions, and power-law parameters τ for different assumptions on the symmetry energy.

The results for $\gamma = 8$ MeV are only slightly different from those obtained for the standard value $\gamma = 25$ MeV.

One can see a small decrease of T , therefore, the average characteristics of produced hot fragments are not very sensitive to the symmetry energy. The main effect of the symmetry energy is manifested in mass variances of the hot fragments (isotope distribution).

Influence of the symmetry energy on fragment partitions-ALADIN

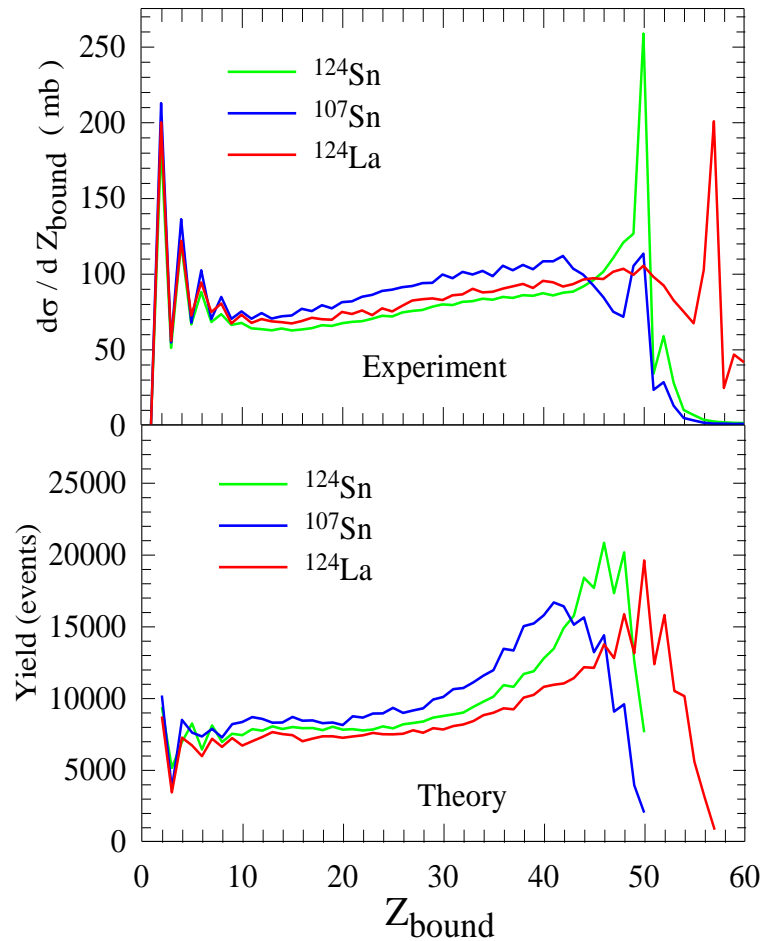


Fig.1. Top panel shows the triggered reaction cross section as a function of Z_b for the three projectiles as indicated. Bottom panel shows the corresponding distributions as obtained from SMM ensemble calculations for 500000 reaction events with standard parameters.

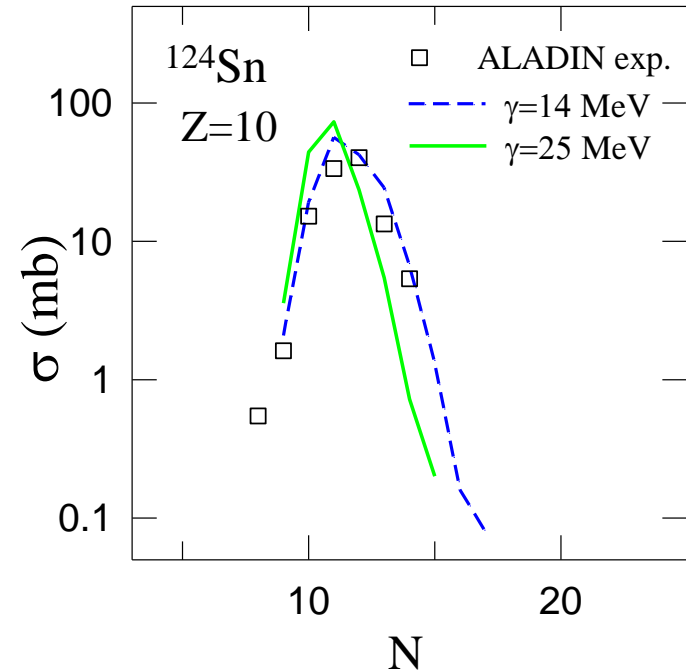


Fig.2. Isotope distribution for ^{124}Sn , empty squares show the ALADIN experimental data. SMM ensemble calculations were globally normalized to the measured Z_{bound} cross sections in the interval $10 < Z_b \leq 30$ for which agreement is the best (see Fig.1), the obtained normalization factor is 0.00804 mb per theoretical event for ^{124}Sn projectile.

Isotopic Distributions of Fragments- ALADIN

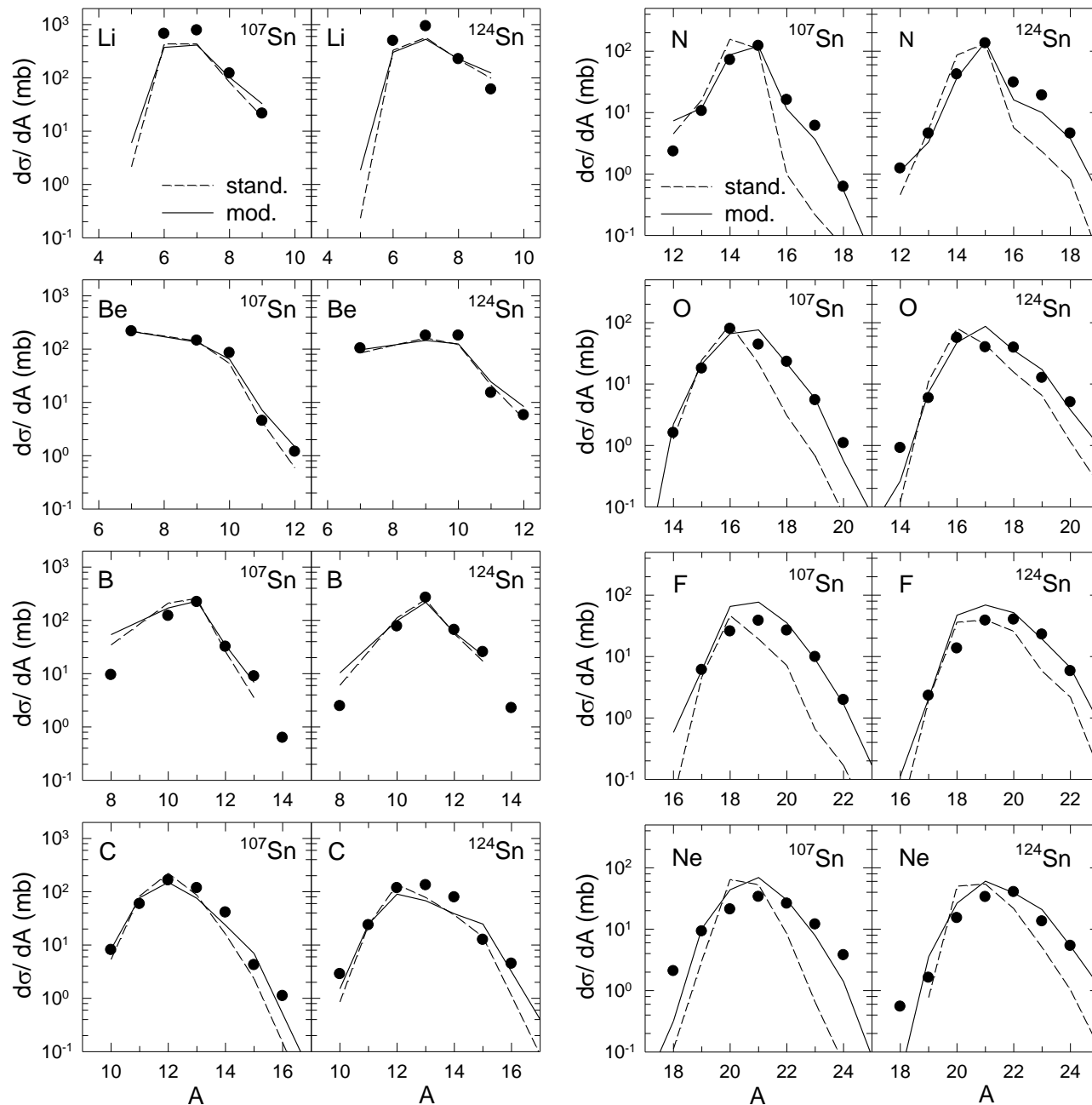


Fig.3. Full circles show experimental data, solid lines for $\gamma = 14$ MeV and dashed lines for ground state value $\gamma = 25$ MeV .

Symmetry energy effects:
It is possible to extract information of the symmetry energy at low densities from which the fragments are formed.

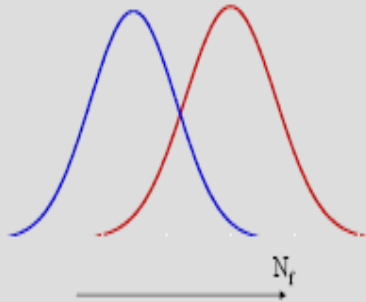
Similar analysis with ensemble Calculations for ALADIN data. Experimental values are reproduced around $\gamma = 14$ MeV.

R. Ogul, A.S. Botvina et al.,
Phys. Rev. C 83, 024608
(2011),

Isoscaling in Nuclear Multifragmentation-ALADIN

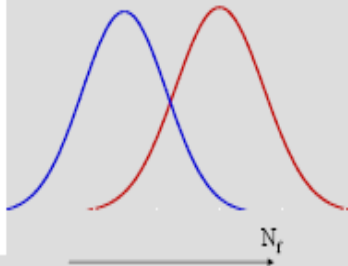
Statistical fragment formation

take two systems
(A_1, Z_1) and (A_2, Z_2)
one neutron poor
one neutron rich



Statistical fragment formation

take two systems
(A_1, Z_1) and (A_2, Z_2)
measure yield ratios
 $R_{21}(N_f, Z_f)$ of fragments
 $\ln(R_{21})$ linear with N_f



basic idea of isoscaling: coefficient reflects widths

Isoscaling has been observed over a variety of reactions including evaporation, strongly damped binary collision, and nuclear multifragmentation.

It is seen that ratios of isotope yields emitted from two reactions exhibit an exponential dependence on the neutron and proton number of the isotope. Let Y_2 denotes the yield of **neutron rich** and Y_1 denotes the yield of **neutron poor** elements. Then the isoscaling ratio is defined as follows:

$$R_{21} = \frac{Y_2(N, Z)}{Y_1(N, Z)} = C \exp(\alpha N + \beta Z)$$

where $R_{21}(N, Z)$ is known as the isoscaling ratio, and the $Y_2(N, Z)$ denotes the yield of the fragments of neutron rich element while $Y_1(N, Z)$ denotes the yield of the fragments of neutron poor element. Here C , α and β are fitting parameters. Experimentally and theoretically approximated formula:

$$\alpha T \approx -4 \gamma (Z_1^2/A_1^2 - Z_2^2/A_2^2)$$

T is known, then symmetry energy coefficient γ can be obtained when isoscaling coefficient α is calculated.

Isoscaling- ALADIN

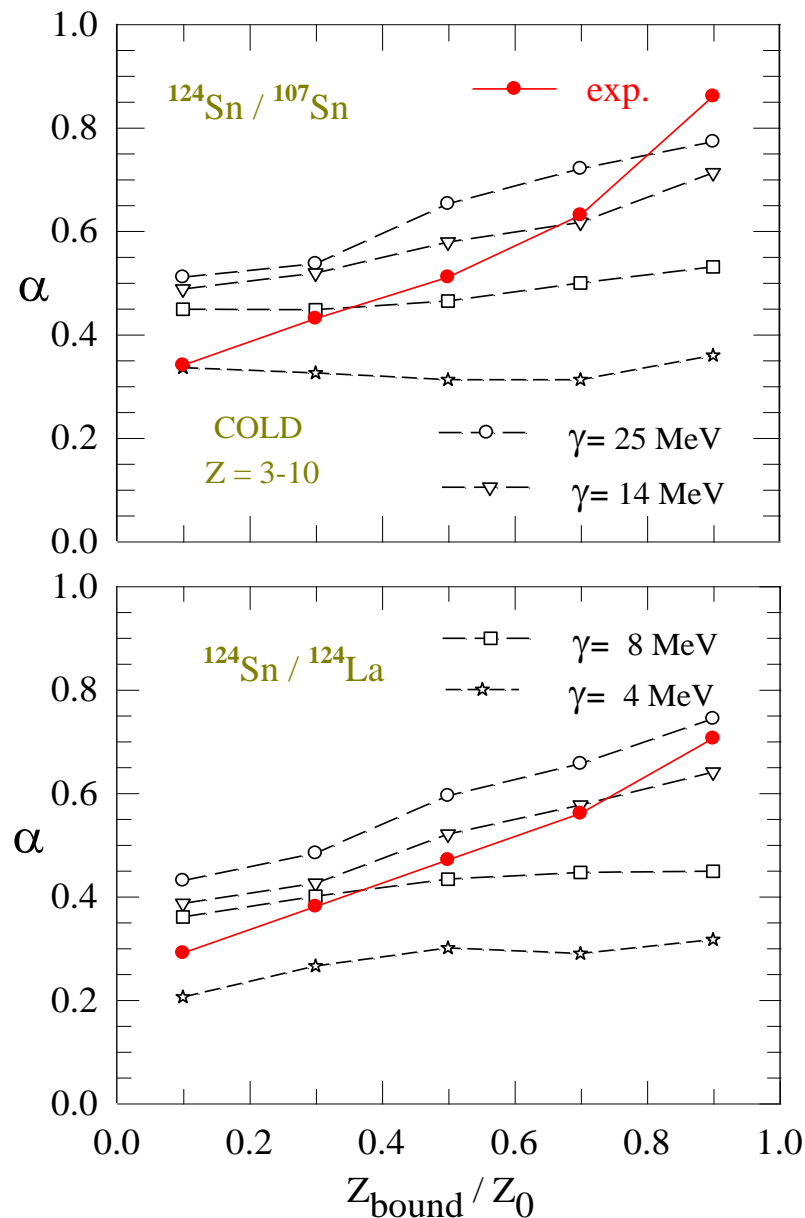
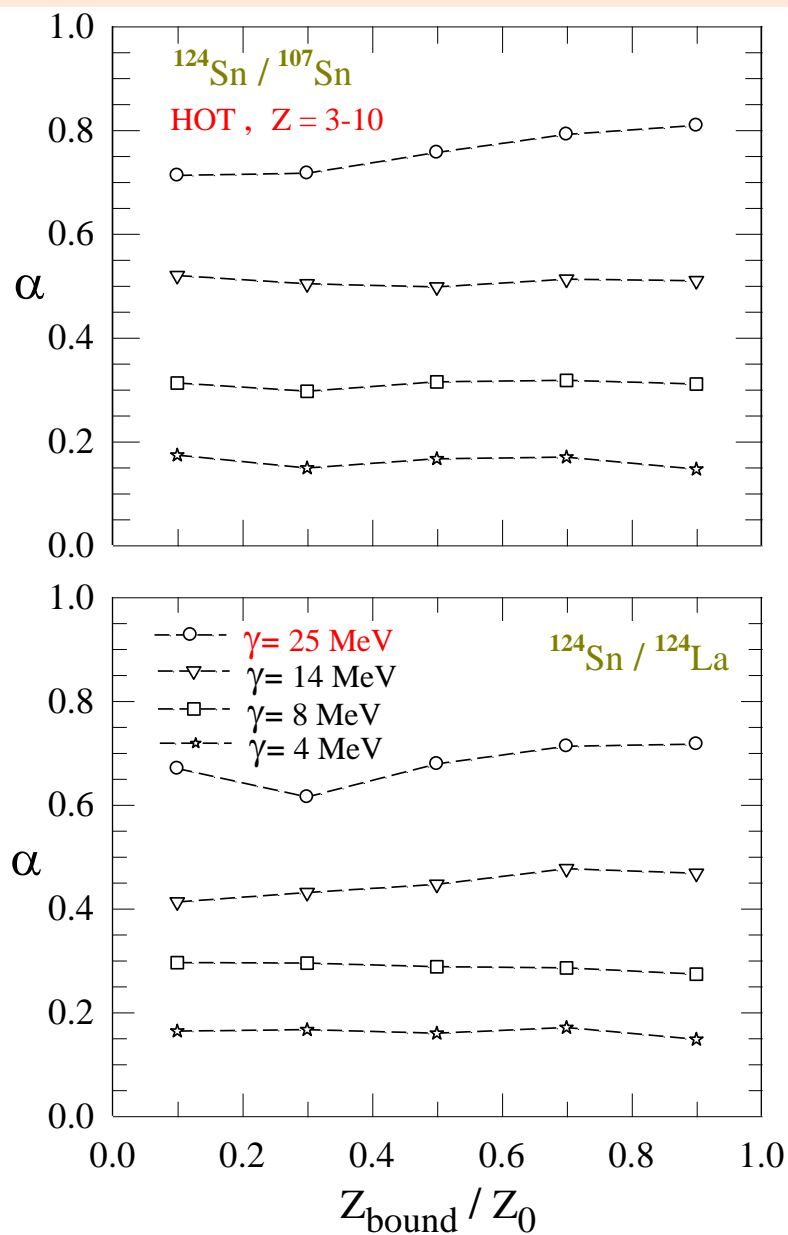


Fig.4. Isoscaling coefficients for hot and cold fragmentations (SMM) and experimental values (ALADIN)

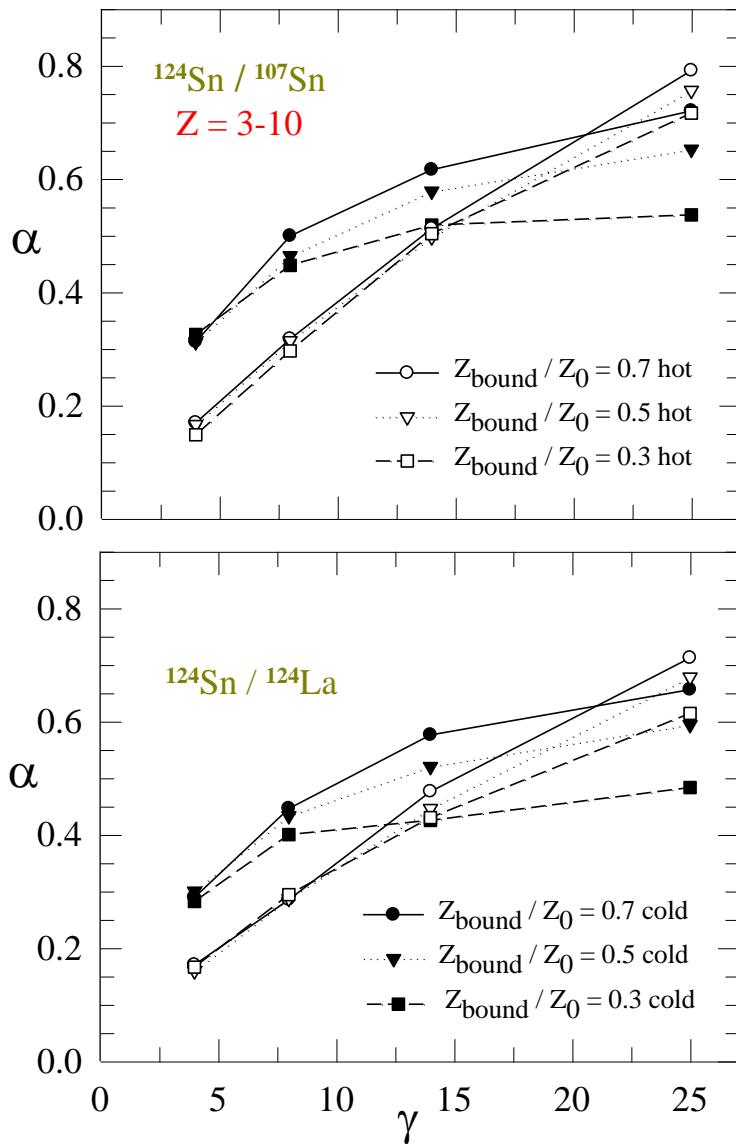


Fig.5. Variation of isoscaling coefficient (alpha) values with symmetry energy.

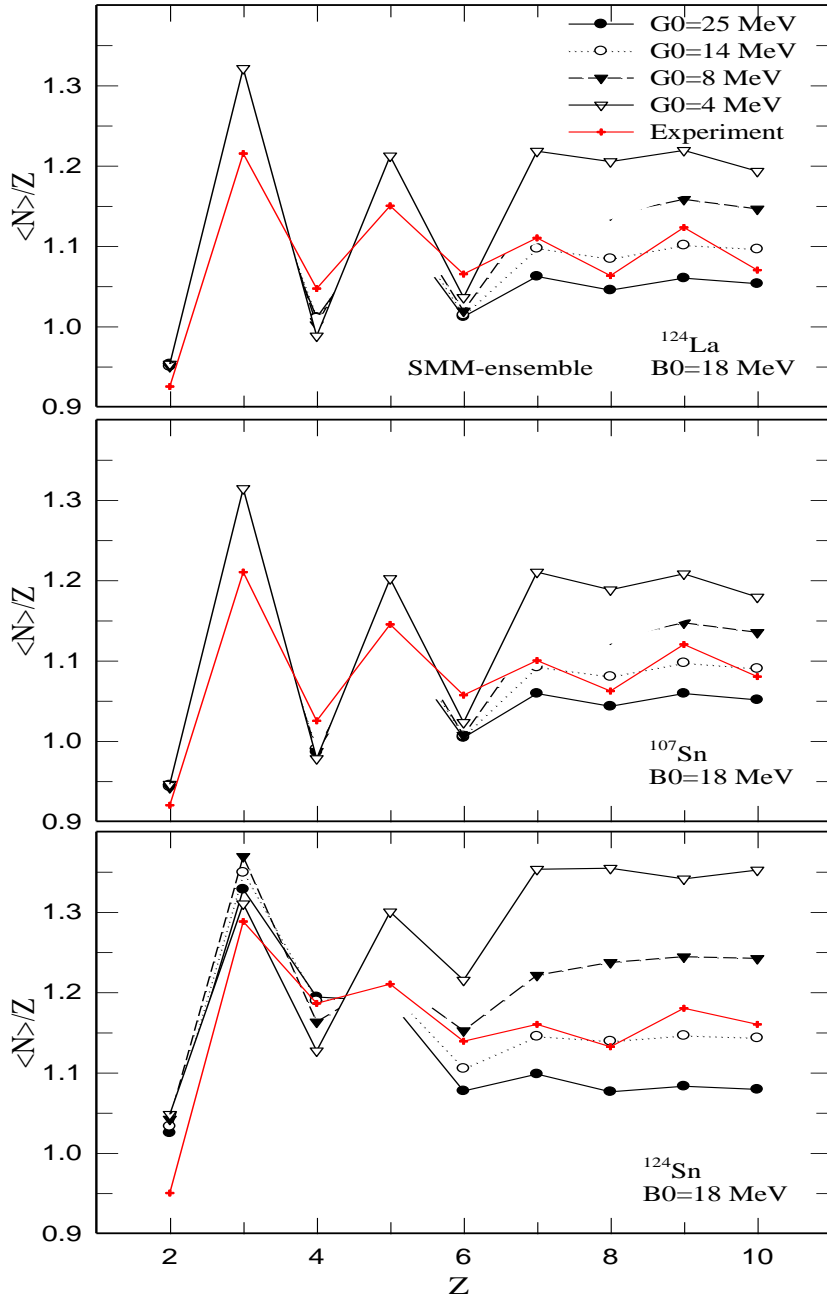
Hot fragments exhibit the linear relation of isoscaling parameters α and γ as expected. With $\gamma=25$ MeV, the sequential processes cause a slight broadening of the isotopic distributions and the resulting alpha is lowered by 10% to 20%, similar to what was reported [1,2].

Conclusion : Symmetry energy parameter γ may be very smaller than ground state value . It has large effect on sequential decay and evaluation of isotopic composition. Our results are consistent with Le Fevre's Markov-chain SMM calculations [1].

[1] A. Le Fevre et al., Phys. Rev. Lett. 94, 162701(2005).

[2] A.S. Botvina, O.V. Lozhkin and W. Trautmann, Phys. Rev. C 65, 044610 (2002).

N/Z calculations – ALADIN



$E/A=600$ MeV/A, projectiles Sn^{124} , Sn^{107} , La^{124} stable&radioac sec. beams produced at FRS and measured at ALADIN-GSI at Schwerionen Synchrotron (SIS). ($Z \leq 10$ fragments.)

To reproduce the ALADIN data symmetry energy term is reduced to somewhere around $\gamma = 14$ MeV.

Fig.6. ALADIN-GSI Data and SMM-Ensemble Calculations: Variations of N/Z values with charge number Z of the fragments for the multifragmentation.

R. Ogul, A.S. Botvina et al., *Phys. Rev. C* 83, 024608 (2011).

Conclusion of the theoretical analyses of experiments

It is confirmed that a significant **reduction of the symmetry term** is found necessary to reproduce the data for light particles ($Z \leq 10$) by means of (a) isotopic curves, (b) isoscaling (c) N/Z analyses.

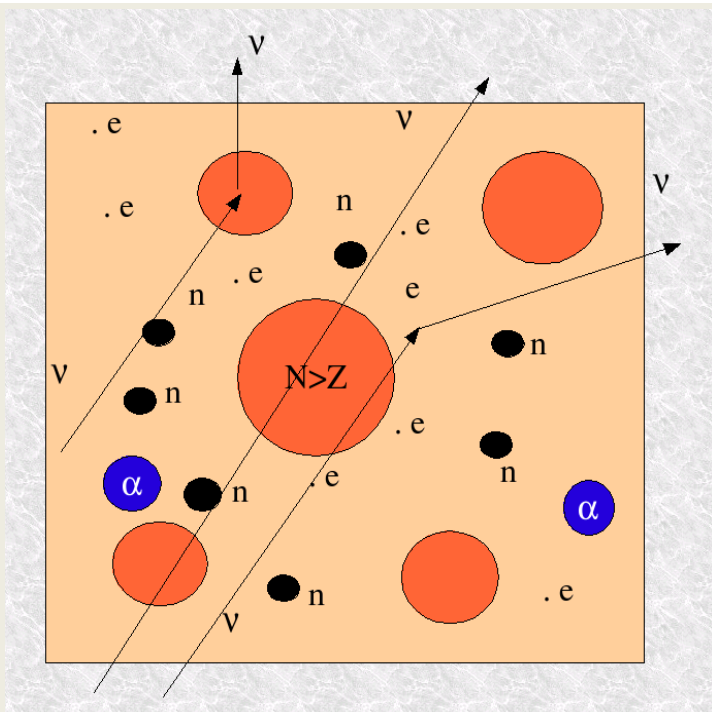
To reproduce the experimental data symmetry energy term is reduced to somewhere around $\gamma = 14 \text{ MeV}$, (ground state value $\gamma = 25 \text{ MeV}$).

This may be related to the details of nuclear structure and properties of such neutron rich nuclei in the vicinity of the **neutron drip-line**.

Variations of isotope distributions with symmetry energy follow the same trend for both ^{112}Sn and ^{124}Sn , but at different scales, so **we need further experiments** to clarify the isospin dependence of the symmetry energy parameter of excited nuclei.

Extracted information from the analyses of this kind of experiments will be particularly useful for the studies of exotic nuclei far from stability and for the simulations of the nuclear composition in **supernova explosion** and **the crust of neutron stars**. In particular, the symmetry energy of fragments should be important **for the exotic nuclei** close to the neutron dripline.

Statistical approach adopted for supernova matter



(1) **Electrons** balance positive charge, (2) **photons** change nucl composition via photo-nucl reactions, (3) a strong **neutrino** wind from a protoneutron star. Neutron capture, photo-disintegs, neutron evaporation, e-/e+ and neutrino/antineutrino reactions are taken into account in the assumptions of statistical eq.

Supernova matter described as a mixture of protons, neutrons, electrons, photons and neutrinos in thermal equilibrium. For macroscopic scales grand canonical approx. Each particle i with B_i baryon number, Q_i charge, L_i lepton, characterized with μ_i

$$\mu_i = B_i \mu_B + Q_i \mu_Q + L_i \mu_L$$

Three independent chem potentials are determined from the conservations of total number of baryons, charge and leptons. Similarly, $\mu_{A,Z}$, μ_e , μ_{neutri} corresponding conservation laws for Baryon, Charge, Lepton numbers:

$$\rho_B = B / V = \text{sum} \{ A \rho_{A,Z} \}$$

$$\rho_Q = Q / V = \text{sum} \{ Z \rho_{A,Z} - \rho_e \} = 0 \quad \text{electrically neutral volume}$$

$$\rho_L = \rho_e + \rho_{\text{neutri}} - \rho_{\text{antineutri}}, \quad \text{with } \rho_e = \rho_{e^-} - \rho_{e^+}$$

The lepton number conservation is a valid concept only if neutrinos and antineutrinos are trapped in the system within the neutrinosphere, otherwise $\mu_L = 0$. If escapes freely from stars then only conservation of B_i and Q_i and Y_e is fixed.

Statistical ensemble with precise values of T , ρ_B , Y_e and fixed value of $A=1000$ baryons in a box density of fragments is fixed at $\rho_0=0.15 \text{ fm}^3$, non-fixed composition of uncertain particle number.

All system is divided to Wigner-Seitz cells containing one nucleus, neutrons and electrons.

$$1 \leq A \leq 1000, \quad 0 \leq Z \leq A$$

Since weak reacts are slow, consider out of equilibrium lepton fraction $Y_L = \rho_L / \rho_B$, or by the electron fraction $Y_e = \rho_e / \rho_B$.

Statistical Model for Supernova Matter (SMSM)

A.S. Botvina, I.N. Mishustin, NPA 843, 98 (2010)

The total free energy density:
$$f = \frac{1}{V} \sum_{AZ} N_{AZ} \left[-T \cdot \ln \left(\frac{g_{AZ}^0 V_f A^{3/2}}{\rho_{AZ} V \lambda_T^3} \right) + 1 \right] + F_{AZ}$$

The nucleon thermal wavelength:
$$\lambda_T = (2\pi\hbar^2 / m_N T)^{1/2} \quad m_N = 939 \text{ MeV}$$

Excluded volume correction:
$$V_f / V = (1 - \rho_B / \rho_0) \quad \rho_B < 0.1 \rho_0$$

$$f = \frac{1}{V} \sum_{AZ} N_{AZ} \left\{ F_{AZ}^{t0} + \ln(V_f / V) + F_{AZ} \right\} \quad F_{AZ}^{t0} = -T \cdot \ln \left(\frac{g_{AZ}^0 V_f A^{3/2}}{N_{AZ} V \lambda_T^3} \right) + 1$$

$$\langle N_{AZ} \rangle = \frac{g_{AZ}^0 V_f A^{3/2}}{\lambda_T^3} \exp \left[\frac{1}{T} (F_{AZ} - \mu_{AZ}) \right]$$

Internal free energy of species (A,Z) (A>4):
$$F_{AZ}(T, \rho) = F_{AZ}^B + F_{AZ}^S + F_{AZ}^C + F_{AZ}^{\text{sym}}$$

Bulk energy:
$$F_{AZ}^B(T) = \left(-w_0 - \frac{T^2}{\varepsilon_0} \right) A, \quad \omega_0 = 16 \text{ MeV}, \quad \varepsilon_0 = 16 \text{ MeV}$$

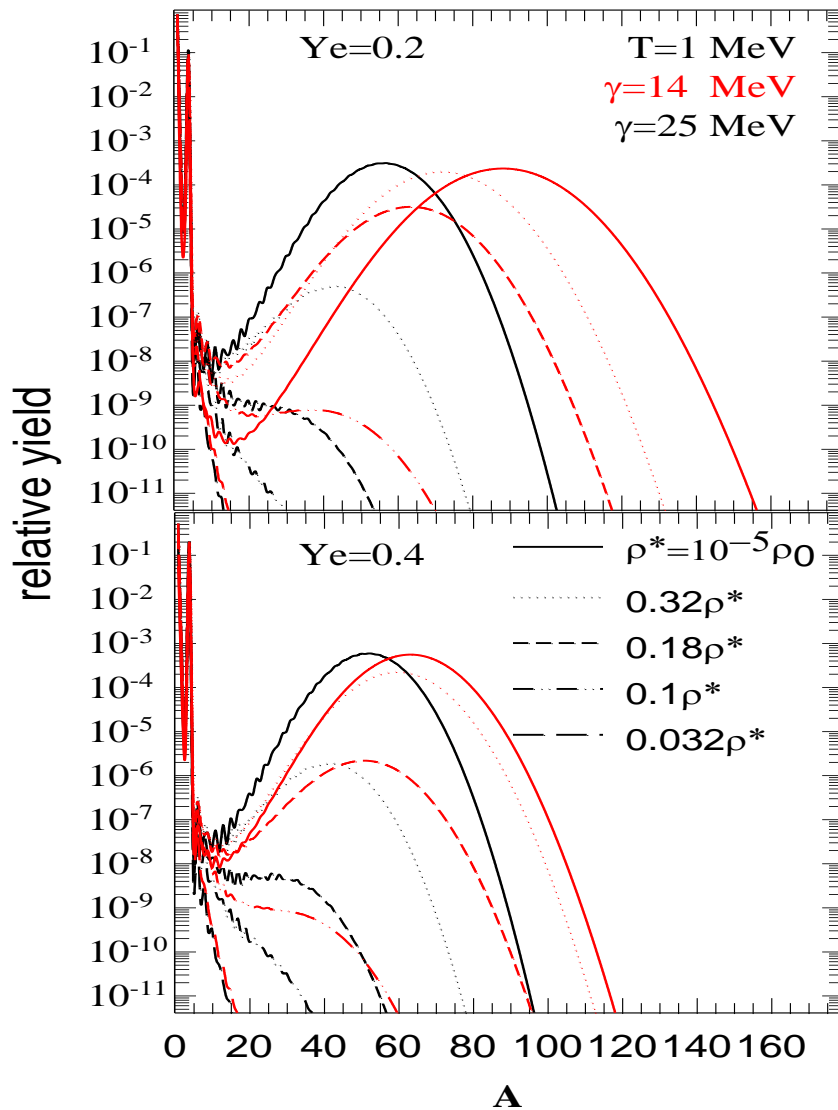
Surface energy:
$$F_{AZ}^S(T) = \beta_0 \left(\frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4} A^{2/3} \quad \beta_0 = 18 \text{ MeV}, \quad T_c = 18 \text{ MeV}$$

Coulomb energy:
$$F_{AZ}^C(\rho) = \frac{3}{5} c(\rho) \frac{(eZ)^2}{r_0 A^{1/3}} \quad c(\rho) = \left[1 - \frac{3}{2} \left(\frac{\rho_e}{\rho_{0p}} \right)^{1/3} + \frac{1}{2} \left(\frac{\rho_e}{\rho_{0p}} \right) \right]$$

Symmetry energy:
$$F_{AZ}^{\text{sym}} = \gamma \frac{(A - 2Z)^2}{A} \quad \gamma = 25 \text{ MeV}$$

Nucleons and light fragments $A \leq 4$
$$F_{AZ} = -B_{AZ} + F_{AZ}^C$$

Influence of the symmetry energy term γ in stellar matter



The symmetry energy coefficient γ is defined by

$$F_{AZ} = \gamma (N-Z)^2 / A$$

Fig.1. Mass distribution of nuclear fragments (yields per nucleon) produced in stellar matter with electron fraction $Y_e = 0.2$ and 0.4 , $T = 1$ and 4 MeV, and several densities, $\rho = 10^{-5} \rho_0$.

As one clearly see, the mass distribution evolves from 'U-shape' at densities (in units $\rho^* = 10^{-5} \rho_0$) the power-law A^{-T} , exponential distribution of intermediate mass fragments ($4 < A < 53$) is more flat.

With decreasing symmetry energy term the mass distributions shift to the neutron rich side and much more heavy nuclei appear.

Here the nuclear system is fully characterized by the temperature T , density ρ , and electron fraction Y_e .

SMSM: EOS of stellar matter

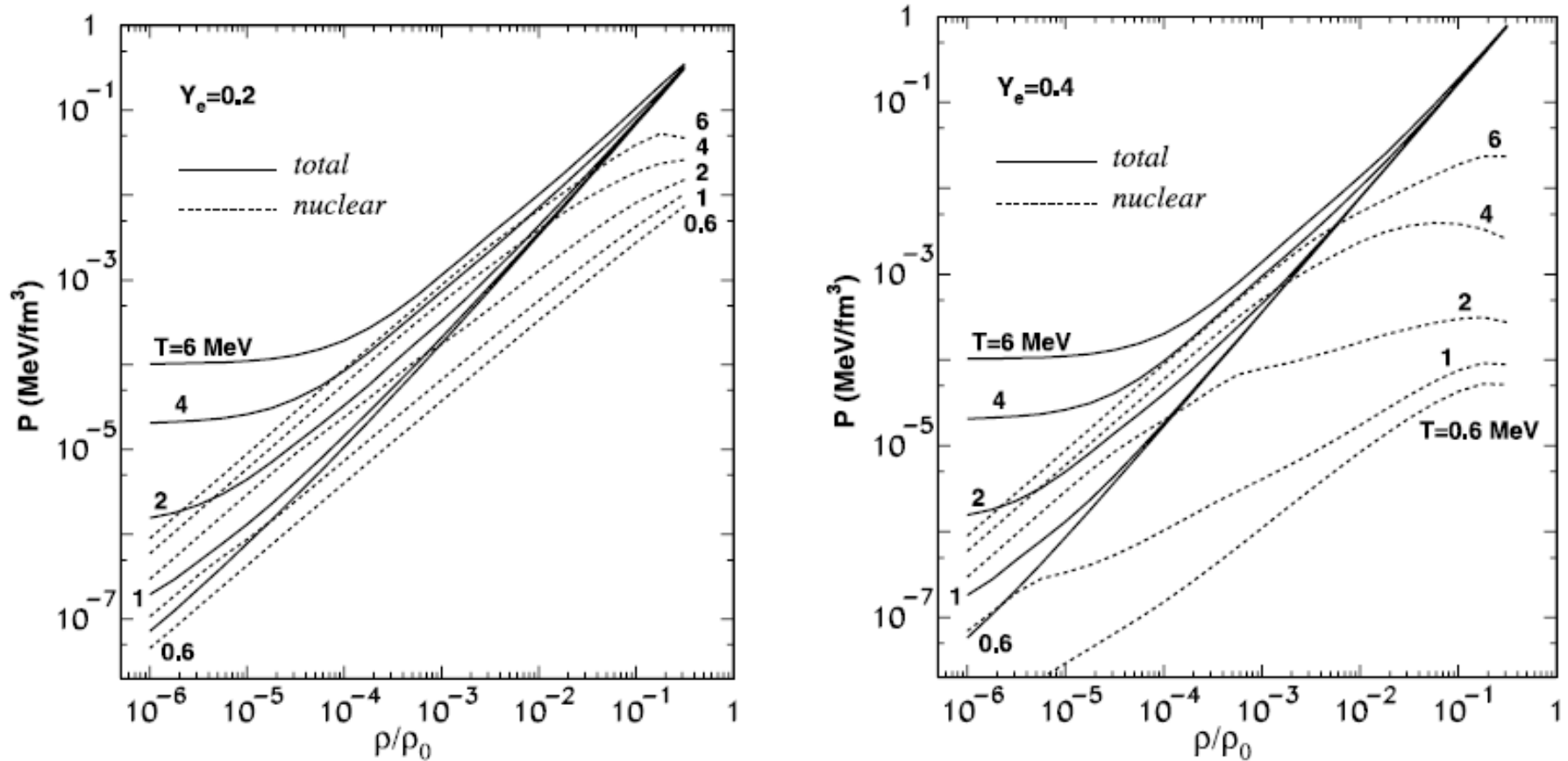


Fig.2. Pressure isotherms as functions of baryon density for $Y_e=0.2$ and 0.4 . Dashed lines are only nuclear contributions, solid lines are total pressures including also electron and photon contributions. *Electrons control the pressure in stellar matter.

Electron capture rate

Reduction of the symmetry energy of hot nuclei can be very important for weak processes. Here, consider a typical example related to deleptonization of matter associated with the electron capture by nuclei.

One should bear in mind that the electron fraction is crucial for dynamics of stellar matter, since the electron pressure dominates at subnuclear densities.

The electron capture rate R_e is calculated by following the method suggested in [1]. It is based on an independent particle model and assumes dominance of Gamow–Teller transitions.

The electron chemical potential μ_e and the reaction Q -value are the most important energy scales of the capture process. It is clear that the Q -value is directly related to the symmetry energy coefficient γ .

[1]. K. Langanke, et al., Phys. Rev. Lett. 90 (2003) 241102.

Symmetry energy effect on electron capture rate

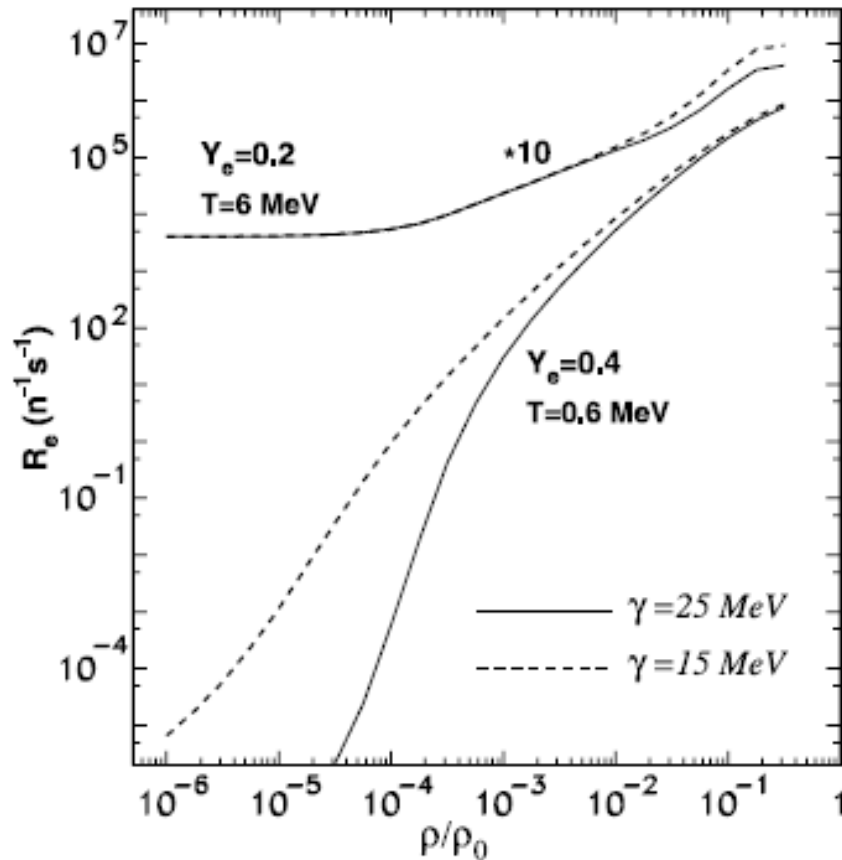


Fig.3. Electron capture rates as a function of the density, (top curves multiplied by 10).

In the first case we see that the reduction of the symmetry energy coefficient γ from 25 to 15 MeV, can enhance the capture rate by about a factor 2. This enhancement is somewhat moderated by a high electron chemical potential, which dominates in the reaction rate. $0.1\rho_0$, $T=6$ MeV, $Y_e=0.2$.

In the case of low densities (less than $10^{-3}\rho_0$) and low temperatures ($T = 0.6$ MeV), the increase of the capture rate with decreasing γ becomes even more dramatic and can reach several orders of magnitude. The reason is that the electron chemical potential is small. In this case the reaction rate is determined by the symmetry energy of nuclei.

Conclusions

The statistical multifragmentation model for nuclear reactions is generalized for the astrophysical processes at subnuclear densities with reasonable assumptions for equilibrium conditions.

The parameters obtained for **nuclear matter** from analyses of experiments can be applied for **stellar matter** (astrophysical processes) too.

The **symmetry energy** of fragments, has significant effects on **stellar matter** properties (fragment yields and compositions) at low density freeze-out.

We need more experimental data on nuclei toward the **neutron-rich side** as much as possible, to determine nuclear compositions and modification of properties of nuclei at subsaturation densities.

We show that excited **nuclear matter** created in nuclear reactions in many respects is similar to **supernova matter**.

It can be applied for a broad variety of **stellar processes**, including the collapse of massive stars and **supernova explosions**, clusterization of nuclear matter in the **crust of neutron stars**, nuclear composition in **merging binary stars**, etc.

Thank you for your participation !